Tentamen Toepassingen van de quantumfysica 5-11-99 1-5

Problem I <u>Hyperfynstructure of Natrium.</u>

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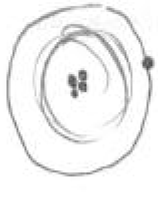


$^{23}Na\,(3s)\,^2S_{1/2}$ $I = 3/2$

$$|\mu_I|_m = g_I \mu_N I$$

$$= 2{,}2161\,\mu_N$$

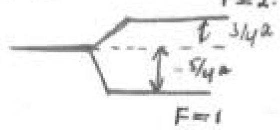
" Je kunt niet van Italianen winnen, alleen verliezen". JC

$$\Rightarrow g_I = 1{,}4774.$$

(a) $\Delta E_{HFS} = \frac{a}{2}\left[F(F+1) - J(J+1) - I(I+1)\right]$ De aantal mogelijke lijnen is gelijk aan het aantal mogelijke F'en

$$F = |I-J|, \ldots, |I+J| \;=\; F = 1, 2 \rightarrow 2 \text{ mogelijke lijnen}$$

(b) $F = 1, \ldots, 2$



$F=1$. $\Delta E_{HFS} = \frac{a}{2}\left[2 - \frac{3}{4} - \frac{15}{4}\right] = -\frac{5}{4}a$

$F=2$ $\Delta E_{HFS} = -\frac{5}{4}a + 2a = \frac{3}{4}a$

(c) $^{24}Na \rightarrow I = 4$, $g_I = 0{,}423$

Alleen de kern veranderd, de electronen wolk blijft hetzelfde

$F = 3\frac{1}{2}, 4\frac{1}{2} \rightarrow 2$ levels.

(d) $a = \dfrac{g_I \mu_N B_J}{\sqrt{J(J+1)}}$, opsplitsing van natrium = 1772 MHz

$$\Delta E_{F+1,F} = a(F+1) \begin{cases} a_1 \cdot 2 \\ a_2 \cdot 4\frac{1}{2} \end{cases}$$

de verschillende a'tjes verschillen op een factor g_I na,

dus $a_2 = \dfrac{0{,}423}{1{,}4774}\, a_1$

dus $\Delta E(^{24}Na) = \underset{\uparrow\,(F+1)}{4\frac{1}{2}} \cdot \underset{\uparrow\,a_2}{\dfrac{0{,}423}{1{,}4774}\, a_1} = \dfrac{1}{2}\left\{4\frac{1}{2}\,\dfrac{0{,}423}{1{,}4774}\right\} \underset{\uparrow\,1772\text{ MHz}}{\Delta E(^{23}Na)}$

Algemeen $\nu = \dfrac{g_I^{\,Nieuw}}{g_I^{\,oud}} \left\{\dfrac{(F+1)^{Nieuw}}{(F+1)^{oud}}\right\} \nu_0$

99% goed.

$\Delta\nu = 1102$ MHz

Problem 2 Decelerating atoms with circularly polarized light in a magn-field

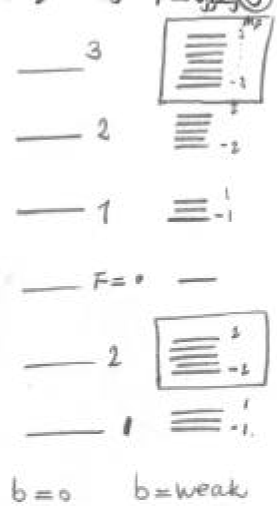
^{87}Rb ; $I = 3/2$

$\lambda = 780$ nm.

$5S_{1/2}$ $J = 1/2 \Rightarrow F = 1, 2$ $g_F = 1/2$

$5P_{3/2}$ $J = 3/2 \Rightarrow F = 0, 1, 2, 3$ $g_F = 2/3$

(a) $5P_{3/2}$



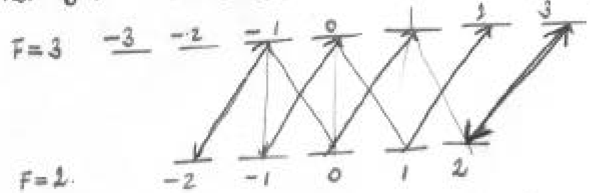
$5S_{1/2}$

$\Delta E_{HFS} = g_F \mu_B m_F B_0$ zwak veld

$\Delta E_{HFS} = g_J \mu_B m_J B_0 + a m_I m_J - g_I \mu_N m_I B_0$ sterk veld.

(b) Door de dopplerverschuiving zal de frequentie hoger zijn. Doordat v verandert moet het B-veld ook steeds veranderen → inhomogeen.

(c)



σ^+ licht, met alleen overgangen $\Delta m_F = +1$.

terugvallen via $\Delta m_F = 0, \pm 1$.

Netto zullen de overgangen omhooggeschoven worden. Daardoor zal de laatste overgang het belangrijkste zijn

↳ $F=3, m_F = 3 \rightarrow F=2, m_F = 2.$ → 2 level systeem.

(d) $\Delta E = h\nu_0 + \mu_B B_0 \{ 3 \cdot \frac{2}{3} - 1 \} = h\nu_0 + \mu_B \cdot B_0$ → de verandering

(e) Alleen absorptieemissie, op een foton komt een frequentie ν aan.

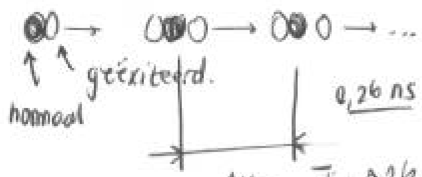
impuls van 1 foton ⇒ $\lambda = \frac{h}{p}$: $\Delta p = \lambda^{-1} h = \frac{h\nu}{c} = \frac{h}{c}\nu_0 (1 + \frac{v}{c})$ ← 10^{-6}!

$$\Delta p \cong \frac{h\nu_0}{c} = \frac{h}{\lambda_0}$$

$$(\Delta p)_{tot} = (m \Delta v) = 87 \cdot 1{,}67 \cdot 10^{-27} \underset{\uparrow\ 300\ m/s}{(\Delta v)_{TOT}} = 4{,}36 \cdot 10^{-23}$$

totaal aantal stappes = $4{,}36 \cdot 10^{-23} / h/\lambda_0 \cong$ 50.000. 99% goed

Vervolg opgave 2.

(f) 

dit gebeurt 50.000 maal.

$$\Delta x = \bar{v} \cdot 0{,}26 \cdot 10^{-9}$$

$$x = 0{,}26 \text{ ns} \left\{ (500-\Delta v) + (500-2\Delta v) + (500-3\Delta v) + \dots + (500-50.000\,\Delta v) \right\}$$

$$= 50.000 \cdot 500 \cdot 0{,}26 \cdot 10^{-9} - \sum_{n=1}^{50.000} n \cdot 0{,}26\,\Delta v = 6{,}27 \cdot 10^{-3} - \frac{(50.000)^2\, 0{,}26 \cdot 10^{-9}}{2}$$

$$\left(1+2+3+\dots+10+\dots n = \frac{n(n+1)}{2}\right)$$

totale lengte zeemanslower $\Delta = 3{,}5$ mm.

(g) $\Delta E = h\nu_0 + \mu_B B_0 = h\nu_0 (1 + v/c)$

$$B_0 = \frac{h\nu_0 v}{\mu_B \cdot c}$$

$$B_0 = \frac{h\nu_0 \sqrt{v_0^2 - 2ax}}{\mu_B c}$$

$$x = \tfrac{1}{2} a t^2 + v_0 t$$

$$a = \frac{\Delta v}{\Delta t} = \frac{0{,}006 \text{ m/s}}{0{,}26 \text{ s}}$$

$$v = -at + v_0 \qquad t = \frac{v_0 - v}{a}$$

$$x = \tfrac{1}{2} a \left(\frac{v_0 - v}{a}\right)^2 + \frac{v_0 (v_0 - v)}{a}$$

$$= -\frac{1}{2a}\left(v_0^2 + v^2 - 2vv_0\right) + \frac{v_0^2}{a} - \frac{vv_0}{a}$$

$$= \frac{v_0^2}{2a} - \frac{1}{2a} v^2 \Rightarrow$$

$$v = \sqrt{v_0^2 - 2ax}$$

(h) wat gebeurt er als we σ^- licht gebruiken

$\cancel{\Delta E = h\nu}$ $\Delta m_F = -1$.

$$\Delta E = h\nu_0 - \mu_B B_0 = h\nu_0 (1 + v/c)$$

Wanneer het magneetveld 0 is en de ~~atoo~~ deeltjes staan daar stil. en worden te de slower weer ingetropt.

99° goed.

Tentamen Toepassingen vd Quantumfysica 5-11-99 blad 4-5

Problem 3 Electron screened coulomb potential

	S	P	D
Δ	0,4115	0,04	0,001.

Li (z=3)

(a) 2s → 2p & 3p

(2p) $\Delta E = -13{,}6 \cdot 9 \cdot \left(\frac{1}{(2-0{,}4115)^2} - \frac{1}{(2-0{,}04)^2} \right) =$ Doe dit

(3p) $\Delta E = +13{,}6\, 9 \cdot \left(\frac{1}{(2-0{,}4115)^2} - \frac{1}{(3-0{,}04)^2} \right) =$ zelf maar!.

(b) $E_n = -13{,}6 \frac{Z^2_{eff}}{n^2} = -13{,}6 \frac{Z^2}{(n-\Delta n)^2}$

$$Z_{eff} = \frac{Zn}{(n-\Delta n)}$$

(c) Grotere n, verder bij de kern vandaan, dus kleinere lading

(d) $1s^2 2p \rightarrow 1s2p$ $\left.\begin{array}{l} l_1 = 0 \\ l_2 = 1 \end{array}\right\}$ $\begin{array}{l} l = \sum_i l_i = L \\ S = 1, 0 \end{array}$

$${}^1P_1,\ {}^3P_{0,1,2}$$

(e) Hund: Grotere spin lagere energie ${}^3P_{0,1,2}$ ${}^1P_1$

(f) $\left.\begin{array}{l} l_1 = 1 \\ l_2 = 0 \end{array}\right\} L = 0,1,2$

$\left.\begin{array}{l} s_1 = 1/2 \\ s_2 = 1/2 \end{array}\right\} S = 1, 0$

voor 2 identieke electronen
moet $l+s$ even zijn:

$${}^1S_0 \quad {}^3P_{0,\cancel{1},1,2} \quad {}^1D_2$$

$${}^3P_0 \quad {}^3P_1 \quad {}^3P_2 \quad {}^1D_2 \quad {}^1S_0$$

vgl hund: hoogste S
hoogste l.

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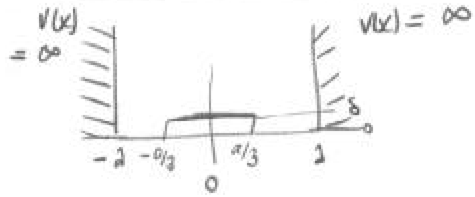
Problem 4 Peturbation



(a) neem $\delta = 0$: $\phi'' = -k^2 \phi$ met $k = \sqrt{2mE}/\hbar$

2 opl: $\phi = A \sin kx$, $\phi = B \cos kx$ $\Big\}$ $\Phi = \sum_k' a_k \sin kx + b_k \cos kx$

$k = n\pi/a$; $k = (n + 1/2)\pi/a$.

(b) Invullen levert op: $E = \dfrac{n^2 \pi^2 \hbar^2}{2ma^2}$ (**)

$E = \dfrac{(n+1/2)\pi \hbar^2}{2ma^2}$ (*)

normalisatie $\frac{1}{\sqrt{a}}$ $\quad \int_{-a}^{a} |\psi|^2 dx = 1.$

dus opl zijn $\phi_0 = \frac{1}{\sqrt{a}} \cos\left(\frac{\pi x}{2a}\right)$ $(n=1)$ (*) $\frac{1}{8} \frac{\pi \hbar^2}{ma^2}$

$\phi_1 = \frac{1}{\sqrt{a}} \sin\left(\frac{\pi x}{a}\right)$ $(n=1)$ (**) $\frac{\pi \hbar^2}{2ma^2}$.

(c) neem nu $\delta \neq 0$ We hebben te maken met een storing klein is t.o.v. de energieën
De storing is te klein om de 2e orde toe te passen.

(d) $E = E^0 + E^1$

$E_0^1 = \langle \phi_0 | H' | \phi_0 \rangle = \frac{\delta}{a} \int_{-a/3}^{a/3} \frac{1}{2} + \frac{1}{2}\left(\cos \frac{2\pi x}{2a}\right) dx = \frac{1}{3}\delta - \frac{\delta}{\pi}\left(\sin \frac{\pi x}{a}\right|_{-a/3}^{a/3} = \frac{1}{3}\delta - \frac{\delta}{\pi}\sqrt{3}$

grondstate

$E_1^1 = \langle \phi_1 | H' | \phi_1 \rangle = \frac{\delta}{a} \int_{-a/3}^{a/3} \frac{1}{2} - \frac{1}{2}\left(\cos \frac{2\pi x}{a}\right) dx = \frac{1}{3}\delta - \frac{\delta}{\pi}\left(\sin \frac{2\pi x}{a}\right)_{-a/3}^{a/3} = \frac{1}{3}\delta + \frac{\delta}{\pi}\sqrt{3}$

(e) $\psi_0 = \phi_0 + \sum_{n \neq 0} \frac{H_{0n}'}{E_n^0 - E_0^0}$ $= \langle \phi | H' | \phi_n \rangle$

$\sum_{n=1}^{\infty} \frac{\frac{\delta}{a}\int_{-a/3}^{a/3} \cos(\pi x/2a) \cos(n+1/2)\frac{\pi x}{a} dx}{E_n^0 - E_0^0} + \sum_{m=1}^{\infty} \frac{\delta}{a} \cancel{\frac{\int_{-a/3}^{a/3} \cos\frac{\pi x}{2a} \sin \frac{n\pi x}{a} dx}{E_m^0 - E_0^0}}$

oneven functie.

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